

Drmota's (x,y,z) is our (z,S,u).

x is z

y is S

z is u

> **eq:=S=(z+z^2)+(z+z^2)*R+u*z^2*(S+S^2);**

$$eq := S = z + z^2 + (z + z^2) R + u z^2 (S + S^2)$$

> **eq2:=solve(R=u*z^2*S*(1+R),R);**

$$eq2 := -\frac{u z^2 S}{-1 + u z^2 S}$$

> **eq:=subs(R=eq2,eq);**

$$eq := S = z + z^2 - \frac{(z + z^2) u z^2 S}{-1 + u z^2 S} + u z^2 (S + S^2)$$

> **F:=op(2,eq);**

$$F := z + z^2 - \frac{(z + z^2) u z^2 S}{-1 + u z^2 S} + u z^2 (S + S^2)$$

Since there is only one equation, the graph is trivially strongly connected.

The system (2.2), (2.3) is

> **S=F;1=diff(F,S);**

$$S = z + z^2 - \frac{(z + z^2) u z^2 S}{-1 + u z^2 S} + u z^2 (S + S^2)$$

$$1 = -\frac{(z + z^2) u z^2}{-1 + u z^2 S} + \frac{(z + z^2) u^2 z^4 S}{(-1 + u z^2 S)^2} + u z^2 (1 + 2 S)$$

The system is simple by combinatorial considerations (nonzero number of base pairs of saturated structures for sufficiently large n and k of moderate size).

Since the system has 2 equations in 3 variables, it makes sense to consider a solution (z,S) of (u).

The parameters mu and sigma^2 are computed as follows.

1. Eliminate S between (2.2) and (2.3) to get a relation between u and z and then differentiate with respect to u to get the derivative of z at u=1.

> **factor(resultant(numer(S-F), numer(1-diff(F,S)),S));**

$$-u^8 z^{18} (1 + z) (4 u^3 z^7 + 4 u^3 z^6 - 27 z^5 u - 6 z^5 u^2 + z^5 u^3 - 54 z^4 u - 6 u^2 z^4 - 33 z^3 u - 2 z^3 u^2 - 6 u z^2 + 4 z + z u + 4)$$

The factor of interest is the last one.

> **P:=op(-1,%);**

$$P := 4 u^3 z^7 + 4 u^3 z^6 - 27 z^5 u - 6 z^5 u^2 + z^5 u^3 - 54 z^4 u - 6 u^2 z^4 - 33 z^3 u - 2 z^3 u^2 - 6 u z^2 + 4 z + z u + 4$$

> **diff(subs(z=z(u),P),u);**

$$12 u^2 z(u)^7 + 28 u^3 z(u)^6 \left(\frac{d}{du} z(u) \right) + 12 u^2 z(u)^6 + 24 u^3 z(u)^5 \left(\frac{d}{du} z(u) \right) - 135 z(u)^4 u \left(\frac{d}{du} z(u) \right) - 27 z(u)^5 - 30 z(u)^4 u^2 \left(\frac{d}{du} z(u) \right) - 12 z(u)^5 u$$

$$\begin{aligned}
& + 5 z(u)^4 u^3 \left(\frac{d}{du} z(u) \right) + 3 z(u)^5 u^2 - 216 z(u)^3 u \left(\frac{d}{du} z(u) \right) - 54 z(u)^4 \\
& - 12 z(u)^4 u - 24 u^2 z(u)^3 \left(\frac{d}{du} z(u) \right) - 99 z(u)^2 u \left(\frac{d}{du} z(u) \right) - 33 z(u)^3 \\
& - 6 z(u)^2 u^2 \left(\frac{d}{du} z(u) \right) - 4 z(u)^3 u - 6 z(u)^2 - 12 u z(u) \left(\frac{d}{du} z(u) \right) \\
& + 4 \left(\frac{d}{du} z(u) \right) + \left(\frac{d}{du} z(u) \right) u + z(u)
\end{aligned}$$

> **dzu:=solve(%,diff(z(u),u));**

$$\begin{aligned}
dzu := & - \left(z(u) \left(12 u^2 z(u)^6 + 12 z(u)^5 u^2 - 27 z(u)^4 - 12 z(u)^4 u + 3 u^2 z(u)^4 - 54 z(u)^3 \right. \right. \\
& - 12 z(u)^3 u - 33 z(u)^2 - 4 u z(u)^2 - 6 z(u) + 1 \left. \right) / \left(28 u^3 z(u)^6 + 24 z(u)^5 u^3 \right. \\
& - 135 z(u)^4 u - 30 u^2 z(u)^4 + 5 z(u)^4 u^3 - 216 z(u)^3 u - 24 z(u)^3 u^2 - 99 u z(u)^2 \\
& \left. - 6 z(u)^2 u^2 - 12 z(u) u + 4 + u \right)
\end{aligned}$$

> **mu:=subs(u=1,-dzu/z(u));**

$$\mu := \frac{12 z(1)^6 + 12 z(1)^5 - 36 z(1)^4 - 66 z(1)^3 - 37 z(1)^2 - 6 z(1) + 1}{28 z(1)^6 + 24 z(1)^5 - 160 z(1)^4 - 240 z(1)^3 - 105 z(1)^2 - 12 z(1) + 5}$$

z(1) is the singularity rho.

2. Same for sigma^2

> **diff(subs(z=z(u),P),u,u):**

> **d2zu:=normal(subs(diff(z(u),u)=dzu,solve(%,diff(z(u),u,u))));**

$$\begin{aligned}
d2zu := & \left(2 z(u) \left(4 + u - 107322 u^2 z(u)^6 - 488142 u^2 z(u)^7 - 72 z(u) - 41352 u^3 z(u)^7 \right. \right. \\
& - 8454 u^3 z(u)^6 - 67044 z(u)^5 u - 7728 z(u)^5 u^2 - 312 z(u)^5 u^3 + 3690 z(u)^4 u \\
& + 1476 u^2 z(u)^4 + 2058 z(u)^3 u + 210 z(u)^3 u^2 + 36 u z(u)^2 - 24 z(u) u + 54432 z(u)^5 \\
& + 20292 z(u)^4 + 2976 z(u)^3 - 208 z(u)^2 + 52488 z(u)^7 + 75168 z(u)^6 + 82 z(u)^4 u^3 \\
& - 14 z(u)^2 u^2 - 1693062 u z(u)^7 - 497226 u z(u)^6 + 3156 z(u)^9 u^5 - 1080 z(u)^7 u^4 \\
& - 3431970 z(u)^8 u - 1237824 z(u)^8 u^2 + 329 z(u)^8 u^5 - 232 z(u)^6 u^4 \\
& - 1823634 z(u)^{10} u^2 - 1923912 z(u)^9 u^2 - 56064 z(u)^{15} u^6 - 63760 z(u)^{14} u^6 \\
& + 459432 z(u)^{13} u^4 - 222624 z(u)^{13} u^5 - 39480 z(u)^{13} u^6 + 691932 z(u)^{12} u^4 \\
& + 565824 z(u)^{11} u^4 - 14056 z(u)^{12} u^6 - 966654 z(u)^{11} u^2 + 1446336 z(u)^{11} u^3 \\
& - 2730 z(u)^{11} u^6 + 1490886 z(u)^{12} u^3 + 18240 z(u)^{17} u^7 + 20640 z(u)^{16} u^7 \\
& - 222912 z(u)^{15} u^5 + 12480 z(u)^{15} u^7 - 317664 z(u)^{14} u^5 + 4260 z(u)^{14} u^7 \\
& + 796068 z(u)^{13} u^3 + 780 z(u)^{13} u^7 + 6720 u^7 z(u)^{18} - 60912 u^5 z(u)^{16} \\
& + 172044 u^3 z(u)^{14} - 1535274 z(u)^{11} u - 3446712 z(u)^{10} u - 4374000 z(u)^9 u \\
& - 295245 z(u)^{12} u - 20928 z(u)^{16} u^6 + 127008 z(u)^{14} u^4 - 218700 z(u)^{12} u^2 \\
& - 226 z(u)^{10} u^6 + 60 z(u)^{12} u^7 - 69045 u^5 z(u)^{12} + 4674 u^5 z(u)^{11} + 733062 u^3 z(u)^{10} \\
& \left. + 267528 u^4 z(u)^{10} + 10602 u^5 z(u)^{10} + 121548 u^3 z(u)^9 + 68772 u^4 z(u)^9 \right)
\end{aligned}$$

$$\begin{aligned} & - 65408 u^3 z(u)^8 + 6132 u^4 z(u)^8 + 14580 z(u)^8) \Big/ (28 u^3 z(u)^6 + 24 z(u)^5 u^3 \\ & - 135 z(u)^4 u - 30 u^2 z(u)^4 + 5 z(u)^4 u^3 - 216 z(u)^3 u - 24 z(u)^3 u^2 - 99 u z(u)^2 \\ & - 6 z(u)^2 u^2 - 12 z(u) u + 4 + u)^3 \end{aligned}$$

> sigma2:=normal(subs(u=1,-d2zu/dzu+mu^2+mu));

$$\begin{aligned} \sigma 2 := & \left(2 \left(8 - 141 z(1) - 757953 z(1)^6 - 38577 z(1)^5 + 33435 z(1)^4 + 7221 z(1)^3 - 263 z(1)^2 \right. \right. \\ & - 2952345 z(1)^7 - 7900494 z(1)^9 - 6257955 z(1)^8 - 5263472 z(1)^{10} - 378672 z(1)^{15} \\ & - 129424 z(1)^{14} + 1375728 z(1)^{13} + 2329544 z(1)^{12} - 260304 z(1)^{11} + 26592 z(1)^{17} \\ & - 84528 z(1)^{16} + 9600 z(1)^{18} \Big) \Big/ \Big((28 z(1)^6 + 24 z(1)^5 - 160 z(1)^4 - 240 z(1)^3 \\ & - 105 z(1)^2 - 12 z(1) + 5)^2 (12 z(1)^6 + 12 z(1)^5 - 36 z(1)^4 - 66 z(1)^3 - 37 z(1)^2 \\ & - 6 z(1) + 1) \Big) \end{aligned}$$

3. Test if sigma^2 is regular:

> subs(z(1)=.424687,sigma2);

1.736258756